1. Editor's note

This issue is published while the very the feeling of the nearby Second SPM Workshop is in the air. More than 50 mathematicians are expected to participate in this fascinating event. Regardless whether you could or could not make it to this meeting, you are encouraged to attend the coming BEST meeting (details below), in which a significant part deals with SPM and related areas.

Contributions to the next issue are, as always, welcome.

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2. On Selective screenability and examples of R. Pol

Let $\mathcal{A}$ and $\mathcal{B}$ be families of collections of subsets of the infinite set S. The symbol $\mathsf{S}_c(\mathcal{A}, \mathcal{B})$ denotes the statement: For each sequence $(O_n : n \in \omega)$ of elements of $\mathcal{A}$ there is a sequence $(T_n : n < \omega)$ where for each n the family T_n is pairwise disjoint, and each element of T_n is a subset of some element of O_n, and $\cup_{n<\omega} T_n$ is an element of $\mathcal{B}$. The author introduced this selection principle.

For X a topological space the symbol $\mathcal{O}$ denotes the collection of all open covers of X. The property $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$ was introduced by Addis and Gresham (1978). It is a selective version of Bing's property of screenability (1951). Spaces with property $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$ are traditionally called C-spaces. According to a theorem of Hattori-Yamada, and independently Rohm [4], if X is σ-compact and both X and Y have $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$, then their product has $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$. Consequently, if X is σ-compact and has property $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$, then it has $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$ in all finite powers.

σ-compactness is stronger than the Hurewicz property, which in turn is stronger than the Menger property [5]. For definitions of these properties see [2]. R. Pol [3] showed that in the above product theorem the σ-compactness of X cannot be weakened to the Menger property, by proving in Theorem 1.5 of [3]: Assuming the Continuum Hypothesis, for each natural number $n \geq 1$ there exists a separable metrizable space X such that:

(1) X^{n+1} has the Menger Property, and

(2) X^n is a C-space, but
(3) X^{n+1} is not a C-space.

We prove that the Menger property cannot be strengthened to the Hurewicz property in such examples:

Theorem 2.1. *Let X be a separable metrizable space with property $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$. For each n, if X^n has the Hurewicz property, then X^n has property $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$.*

Thus, if X has property $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$, and if it has the Hurewicz property in all finite powers, then it has the property $\mathsf{S}_c(\mathcal{O}, \mathcal{O})$ in all finite powers.

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Liljana Babinkostova

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3. WORKSHOPS AND CONFERENCES

3.1. **The Oxford Conference on Topology and Computer Science in Honour of Peter Collins and Mike Reed.** 7–10 August 2006, Oxford, UK. This conference will mark the retirements of Peter Collins and Mike Reed from the Oxford Mathematical Institute and the Oxford Computing Laboratory. The conference will take the week before the Prague Symposium and will include special sessions on Set-theoretic Topology, Theoretical Computer Science and Continuum Theory and Dynamics.

3.2. **Boise Extravaganza In Set Theory (BEST2006).** March 31–April 2, 2006, Boise State University Boise, ID, USA.

We are pleased to announce our fifteenth annual BEST conference. There will be four talks by invited speakers: Natasha Dobrinen (Kurt Godel Research Center for Mathematical Logic), Michael Hrusak (UNAM), Istvan Juhasz (Alfred Renyi Institute of Mathematics, Budapest), Boban Velickovic (Equipe de Logique Mathematique, Universite de Paris 7).

The current list of participants consists of Liljana Babinkostova (Boise State University), Randall Holmes (Boise State University), Richard Laver (University of Colorado, Boulder), Justin Moore (Boise State University), Marion Scheepers (Boise State University), and Boaz Tsaban (Weizmann Institute of Science).

If you wish to participate but are not listed above or if you are not on the mailing list for the conference and wish to be, please contact the organizers.

The conference webpage at

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http://diamond.boisestate.edu/ best/best15/best15.html

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contains the most current information including lodging, abstract submission, travel, schedule, etc.

The talks will take up a full day on Friday and Saturday and a half day (the morning) on Sunday. The exact details of the schedule are not yet available. If you wish to participate but will not attend the entire conference, please let us know in advance.

Financial Support: Limited financial support is available for participants - particularly graduate students and young researchers - who do not already have sufficient support to attend the conference. In order to apply, e-mail one of the organizers.

Abstracts (generously provided by Atlas Conferences Inc.)

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http://atlas-conferences.com/cgi-bin/abstract/casb-01

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Organized by Liljana Babinkostova, Stefan Geschke, Justin Moore, and Marion Scheepers. You can reach us by writing to `best@math.boisestate.edu`

4. Research announcements

4.1. **The isometry group of the Urysohn space as a Levy group.** We prove that the isometry group Iso($\mathbb{U}$) of the universal Urysohn metric space $\mathbb{U}$ equipped with the natural Polish topology is a Lévy group in the sense of Gromov and Milman, that is, admits an approximating chain of compact (in fact, finite) subgroups, exhibiting the phenomenon of concentration of measure. This strengthens an earlier result by Vershik stating that Iso($\mathbb{U}$) has a dense locally finite subgroup. We propose a reformulation of Connes' Embedding Conjecture as an approximation-type statement about the unitary group $U(\ell^2)$, and show that in this form the conjecture makes sense also for Iso($\mathbb{U}$).

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http://arxiv.org/abs/math.GN/0509402

Vladimir Pestov
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4.2. **Chasing Silver.** We show that limits of CS iterations of n-Silver forcing notion have the n-localization property.

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http://arxiv.org/abs/math.LO/0509392

Andrzej Roslanowski and Juris Steprans
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4.3. **Disjoint Non-Free Subgoups of Abelian Groups.** Let G be an abelian group and let λ be the smallest rank of any group whose direct sum with a free group is isomorphic to G. If λ is uncountable, then G has λ pairwise disjoint, non-free subgroups. There is an example where λ is countably infinite and G does not have even two disjoint, non-free subgroups.

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http://arxiv.org/abs/math.LO/0509406

Andreas Blass and Saharon Shelah
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4.4. **Characterizing metric spaces whose hyperspaces are absolute neighborhood retracts.** We characterize metric spaces X whose hyperspaces 2^X or $Bd(X)$ of non-empty closed (bounded) subsets, endowed with the Hausdorff metric, are absolute [neighborhood] retracts.

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http://arxiv.org/abs/math.GT/0509395

T. Banakh and R. Voytsitskyy
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4.5. **A Vitali set can be homeomorphic to its complement.** We prove that:
  (1) There exists a special automorphism F of the Cantor space such that every noncancellable composition of finite powers of F and translations of rational numbers has no fixed point.
  (2) For this automorphism there exists both Vitali and Bernstein subset of the Cantor space such that the image by F of this set is equal to its complement.

(3) There exists a Bernstein and Vitali set such that there is no Borel isomorphism between this set and its complement.

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http://delta.univ.gda.pl/~andrzej/vitali.zip

Andrzej Nowik

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4.6. o-Boundedness of free objects over a Tychonoff space.

We characterize various sorts of boundedness of the free (abelian) topological group $F(X)$ ($A(X)$) as well as the free locally-convex linear topological space $L(X)$ in terms of properties of a Tychonoff space X. These properties appear to be close to so-called selection principles, which permits us to show, that (it is consistent with ZFC that) the property of Hurewicz (Menger) is l-invariant. This gives a method of construction of OF-undetermined topological groups with strong combinatorial properties.

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http://arxiv.org/abs/math.GN/0509607

Lyubomyr Zdomskyy

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4.7. On the consistency strength of the Milner-Sauer conjecture.

Motivated by a conjecture of Milner and Sauer regarding partial orders of singular cofinality, we came to prove a theorem about topological spaces of singular density.

We prove that the existence of a topological space $\langle X, O \rangle$ satisfying $d(X) = w(X) = \lambda > \mathsf{cf}(\lambda) \geq \hat{h}(X)$ implies $\mathsf{cf}([\lambda]^{<\mathsf{cf}(\lambda)}, \subseteq) > \lambda$. One consequence is that the existence of a hereditarily Lindelöf topological space of density and weight $\aleph_{\omega_1}$ implies the existence of a measurable cardinal in an inner model of ZFC.

On the way, we also notice that for any topological space $\langle X, O \rangle$, if $d(X)$ is a singular cardinal, then $|O| > d(X)$.

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http://dx.doi.org/10.1016/j.apal.2005.09.012

Assaf Rinot

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4.8. Reconstruction of manifolds and subsets of normed spaces from subgroups of their homeomorphism groups.

We prove various reconstruction theorems about open subsets of normed spaces. E.g. if the uniformly continuous homeomorphism groups of two such sets are isomorphic, then this isomorphism is induced by a uniformly continuous homeomorphism between these open sets.

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http://arxiv.org/abs/math.GN/0510118

Matatyahu Rubin and Yosef Yomdin

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4.9. Reconstruction theorem for homeomorphism groups without small sets and non-shrinking functions of a normed space.

We prove a reconstruction theorem for homeomorphism groups of open sets in metrizable locally convex topological vector spaces. We show that certain small subgroups of the full homeomorphism group obey the conditions of the above theorem.

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http://arxiv.org/abs/math.GN/0510120

Vladimir P. Fonf and Matatyahu Rubin

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4.10. **Locally Moving Groups and the Reconstruction Problem for Chains and Circles.** We prove that complete Boolean algebras can be reconstructed from any locally moving subgroup of their full automorphism group. We use this theorem in order to prove that linear orders and circles can be reconstructed from small subgroups of their full automorphism groups.

http://arxiv.org/abs/math.LO/0510122
Stephen McCleary and Matatyahu Rubin

4.11. **Divisibility of countable metric spaces.** Prompted by a recent question of G. Hjorth as to whether a bounded Urysohn space is indivisible, that is to say has the property that any partition into finitely many pieces has one piece which contains an isometric copy of the space, we answer this question and more generally investigate partitions of countable metric spaces.

We show that an indivisible metric space must be totally Cantor disconnected, which implies in particular that every Urysohn space $\mathbb{U}_V$ with V bounded or not but dense in some initial segment of $\mathbb{R}_+$, is divisible. On the other hand we also show that one can remove "large" pieces from a bounded Urysohn space with the remainder still inducing a copy of this space, providing a certain "measure" of the indivisibility. Associated with every totally Cantor disconnected space is an ultrametric space, and we go on to characterize the countable ultrametric spaces which are homogeneous and indivisible.

http://arxiv.org/abs/math.CO/0510254
Christian Delhomme, Claude Laflamme, Maurice Pouzet, and Norbert Sauer

4.12. **Pre-compact families of finite sets of integers and weakly null sequences in Banach spaces.** We provide a somewhat general framework for studying weakly null sequences in Banach spaces using Ramsey theory of families of finite subsets of integers.

http://arxiv.org/abs/math.FA/0510407
Jordi Lopez Abad and Stevo Todorcevic

4.13. **A semifilter approach to selection principles II: τ^*-covers.** In this paper we settle all questions whether the properties P and Q provably coincide, where P and Q run over selection principles of the type $\mathsf{U}_{fin}(\mathcal{O}, \mathcal{A})$.

http://arxiv.org/abs/math.GN/0510484
Lyubomyr Zdomskyy

4.14. **Parametrizing the abstract Ellentuck theorem.** We give a parametrization with perfect sets of the abstract Ellentuck theorem. The main tool for achieving this goal is a sort of parametrization of an abstract version of the Nash-Williams theorem. As corollaries, we obtain some known classical results like the parametrized version of the Galvin-Prikry theorem due to Miller and Todorcevic, and the parametrized version of Ellentuck's theorem due to Pawlikowski. Also, we obtain parametized vesions of nonclassical results such as Milliken's theorem.

http://arxiv.org/abs/math.LO/0510477
Jose G. Mijares

4.15. **A notion of selective ultrafilter corresponding to topological Ramsey spaces.** We introduce the relation of *almost-reduction* in an arbitrary topological Ramsey space $\mathcal{R}$ as a generalization of the relation of almost-inclusion on $\mathbb{N}^{[\infty]}$. This leads us to a type of ultrafilter $\mathcal{U}$ on the set of first approximations of the elements of $\mathcal{R}$ which corresponds to the well-known notion of *selective ultrafilter* on $\mathbb{N}$. The relationship turns out to be rather exact in the sense that it permits us to lift several well-known facts about selective ultrafilters on $\mathbb{N}$ and the Ellentuck space $\mathbb{N}^{[\infty]}$ to the ultrafilter $\mathcal{U}$ and the Ramsey space $\mathcal{R}$. For example, we prove that the Open Coloring Axiom holds on $L(\mathbb{R})[\mathcal{U}]$, extending therefore the result from [Prisco and Todorcevic, *Perfect-set properties in $L(\mathbb{R})[\mathcal{U}]$*, Adv. Math. **139** (1998), 240-259], which gives the same conclusion for the Ramsey space $\mathbb{N}^{[\infty]}$.

http://arxiv.org/abs/math.LO/0510517

Jose G. Mijares

4.16. **Compact spaces generated by retractions.** We study compact spaces which are obtained from metric compacta by iterating the operation of inverse limit of continuous sequences of retractions. We denote this class by R. Allowing continuous images in the definition of class R, one obtains a strictly larger class, which we denote by RC. We show that every space in class RC is either Corson compact or else contains a copy of the ordinal segment $[0, \aleph_1]$. This improves a result of Kalenda, where the same was proved for the class of continuous images of Valdivia compacta. We prove that spaces in class R do not contain cutting P-points (see the definition below), which provides a tool for finding spaces in RC minus R. Finally, we study linearly ordered spaces in class RC. We prove that scattered linearly ordered compacta belong to RC and we characterize those ones which belong to R. We show that there are only 5 types (up to order isomorphism) of connected linearly ordered spaces in class R and all of them are Valdivia compact. Finally, we find a universal pre-image for the class of all linearly ordered Valdivia compacta.

http://arxiv.org/abs/math.GN/0511567

Wieslaw Kubis

4.17. **Gromov-Hausdorff ultrametric.** We show that there exists a natural counterpart of the Gromov-Hausdorff metric in the class of ultrametric spaces. It is proved, in particular, that the space of all ultrametric spaces whose metric take values in a fixed countable set is homeomorphic to the space of irrationals.

http://arxiv.org/abs/math.GN/0511437

Ihor Zarichnyi

4.18. **Computing the complexity of the relation of isometry between separable Banach spaces.** We compute here the Borel complexity of the relation of isometry between separable Banach spaces, using results of Gao, Kechris and Weaver.

http://arxiv.org/abs/math.FA/0511456

Julien Melleray

4.19. **On some classes of Lindelöf Sigma-spaces.** We consider special subclasses of the class of Lindelöf Sigma-spaces obtained by imposing restrictions on the weight of the elements of compact covers that admit countable networks: A space X is in the class $L\Sigma(\leq \kappa)$ if it admits a cover by compact subspaces of weight κ and a countable network for the cover. We restrict our attention to $\kappa \leq \omega$. In the case $\kappa = \omega$, the class includes the class of metrizably fibered spaces considered by Tkachuk, and the P-approximable spaces considered by Tkacenko. The case $\kappa = 1$ corresponds to the spaces of countable network weight, but even the case $\kappa = 2$ gives rise to a nontrivial class of spaces. The relation of known classes of compact spaces to these classes is considered. It is shown that not every Corson compact of weight $\aleph_1$ is in the class $L\Sigma(\leq \omega)$, answering a question of Tkachuk. As well, we study whether certain compact spaces in $L\Sigma(\leq \omega)$ have dense metrizable subspaces, partially answering a question of Tkacenko. Other interesting results and examples are obtained, and we conclude the paper with a number of open questions.

http://arxiv.org/abs/math.GN/0511606

Wieslaw Kubis, Oleg Okunev, and Paul J. Szeptycki

4.20. **On the depth of Boolean algebras.** We show that the Depth$^+$ of an ultra-product of Boolean algebras, can not jump over the Depth$^+$ of every component by more than one cardinality. We can have, consequently, similar results for the Depth invariant.

http://arxiv.org/abs/math.LO/0512217

Saharon Shelah

5. Problem of the Issue

In the paper

http://arxiv.org/abs/math.GN/0510162

we formulate three problems concerning topological properties of sets generating Borel non-σ-compact groups. In case of the concrete $F_{\sigma\delta}$-subgroup of the Cantor group this gives an equivalent reformulation of the Scheepers diagram problem. The problems are related to the following open problem.

Problem 5.1. *Can a Borel non-σ-compact group be generated by a Hurewicz subspace?*

Lyubomyr Zdomskyy

6. Problems from earlier issues

Issue 1. *Is $\binom{\Omega}{\Gamma} = \binom{\Omega}{\mathrm{T}}$?*

Issue 2. *Is $\mathsf{U}_{fin}(\Gamma, \Omega) = \mathsf{S}_{fin}(\Gamma, \Omega)$? And if not, does $\mathsf{U}_{fin}(\Gamma, \Gamma)$ imply $\mathsf{S}_{fin}(\Gamma, \Omega)$?*

Issue 4. *Does $\mathsf{S}_1(\Omega, \mathrm{T})$ imply $\mathsf{U}_{fin}(\Gamma, \Gamma)$?*

Issue 5. *Is $\mathfrak{p} = \mathfrak{p}^*$? (See the definition of $\mathfrak{p}^*$ in that issue.)*

Issue 6. *Does there exist (in ZFC) an uncountable set satisfying $\mathsf{S}_1(\mathcal{B}_\Gamma, \mathcal{B})$?*

Issue 8. *Does $X \notin \mathsf{NON}(\mathcal{M})$ and $Y \notin \mathsf{D}$ imply that $X \cup Y \notin \mathsf{COF}(\mathcal{M})$?*

Issue 9. *Assume CH. Is $\mathsf{Split}(\Lambda, \Lambda)$ preserved under finite unions?*

Issue 10. *Is $\mathsf{cov}(\mathcal{M}) = \mathfrak{od}$? (See the definition of $\mathfrak{od}$ in that issue.)*

Issue 11. *Does $\mathsf{S}_1(\Gamma, \Gamma)$ always contain an element of cardinality $\mathfrak{b}$?*

Issue 12. *Could there be a Baire metric space M of weight $\aleph_1$ and a partition $\mathcal{U}$ of M into $\aleph_1$ meager sets where for each $\mathcal{U}' \subset \mathcal{U}$, $\bigcup \mathcal{U}'$ has the Baire property in M?*

Issue 14. *Does there exist (in ZFC) a set of reals X of cardinality $\mathfrak{d}$ such that all finite powers of X have Menger's property $\mathsf{U}_{fin}(\mathcal{O}, \mathcal{O})$?*

Issue 15. *Can a Borel non-σ-compact group be generated by a Hurewicz subspace?*

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